Section 2.1

Math 231

Hope College

Properties of Matrix Operations

Theorem 2.4: Throughout this result, matrices A, B, and C are assumed to be $m \times n$ matrices. The symbol $\mathbf{0}$ represents the $m \times n$ zero matrix.

1. For all A, we have $0 \cdot A = \mathbf{0}$.
2. For all A, we have $1 \cdot A = A$.
3. For all $\alpha \in \mathbb{R}$, we have $\alpha \cdot \mathbf{0} = \mathbf{0}$.
4. For all A and B, we have $A + B = B + A$ (commutativity)
5. For all A, B, and C, we have $(A + B) + C = A + (B + C)$ (associativity)
6. For all A and all $\alpha, \beta \in \mathbb{R}$, we have $(\alpha + \beta)A = \alpha A + \beta A$ (distributivity)
7. For all A and B and all $\alpha \in \mathbb{R}$, we have $\alpha(A + B) = \alpha A + \alpha B$ (distributivity)
8. For all A and all $\alpha, \beta \in \mathbb{R}$, we have $(\alpha\beta)A = \alpha(\beta A)$.
9. For all A, we have $A + (-A) = (-A) + A = \mathbf{0}$.

Properties of Matrix Multiplication

Theorem 2.10: Throughout this result, matrices are assumed to be of sizes that can be multiplied or added together. Capital letters (A, B, and C) represent matrices. The symbol I represents an appropriate sized identity matrix.

1. For all A and all $k \geq 1$, we have $A\mathbf{0} = \mathbf{0}$ and $\mathbf{0}A = \mathbf{0}$.
2. For all A, we have $IA = A$ and $AI = A$.
3. For all A, B, and C, we have $A(BC) = (AB)C$. (associativity)
4. For all A, B, and C, we have $A(B + C) = AB + AC$ and $(B + C)A = BA + CA$. (distributivity)
5. For all A and B and all $\alpha \in \mathbb{R}$, we have $A(\alpha B) = \alpha(AB) = (\alpha A)B$.